Communications

Performance Analysis of Derivative Constraint Adaptive Arrays with Pointing Errors

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Abstract—The performance of adaptive arrays with arbitrary order derivative constraints to avoid the signal nulling caused by the error in the steering angle is investigated. The generalized sidelobe canceller (GSC) is used to evaluate the bearing responses of adaptive arrays with pointing errors. Under the assumption of equispaced linear arrays, we utilize the Legendre polynomial to derive a closed-form formula for the sidelobe leakage factor, the ratio in which the signal leaks into the GSC sidelobe cancelling branch. Moreover, we derive the effective beamwidth, the maximum range of pointing inaccuracy allowable to keep the signal loss in array output less than a given value, for the GSC with the delay-and-sum quiescent beamformer. Such kind of adaptive arrays can achieve the maximum output signal-to-noise ratio when the presteering is perfect, and its performance is independent of the location of the phase origin. Numerical results are presented to confirm the analysis.

I. INTRODUCTION

Adaptive arrays with a look-direction constraint [1], [2] are very effective in suppressing interference and can achieve the maximum signal-to-interference-plus-noise ratio (SINR). However, an error in the steering angle, termed the pointing error, will cause the adaptive array to tend to null out the desired signal as if it were a jammer [3]–[6]. Therefore, the “effective beamwidth” of the look-direction constraint adaptive array [3], [6], [7] is much narrower than that of the conventional nonadaptive array. The greater the desired signal power, the worse the signal suppression and the narrower the effective beamwidth. A remedy to this problem is imposing derivative constraints [8]–[10] on the main beam in the look direction. By controlling the first few derivatives of the main lobe, adaptive arrays can avoid placing nulls in the zone of the main lobe. The antijamming performance of the first-order derivative constraint array with perfect pointing has been analyzed in [16]. With pointing error present, [17] examined the output signal-to-noise ratio (SNR) of the first order derivative constraint array by using the technique of polynomial expansion. However, [17] did not analyze the effective beamwidth of the derivative constraint adaptive arrays. In this communication, we will derive a closed-form expression for the normalized array gain of the first order and the higher order derivative constraint arrays with pointing error, from which we can derive the effective beamwidth. Our analysis is performed based on the generalized sidelobe canceller (GSC) [11]–[13], whose performance is identical to the direct-form array with the same constraints. The GSC consists of a sidelobe cancelling branch and a quiescent beamformer [14].

In the presence of pointing error, the signal leaking into the sidelobe cancelling branch causes the cancellation of the signal reflected in the quiescent beamformer [15]. The ratio of the signal sidelobe leakage power to the total signal power received by the array can be termed the sidelobe leakage factor. Under the assumption of equispaced linear arrays, we utilize the theory of Legendre polynomial and derive an explicit formula to predict the sidelobe leakage factors for arbitrary order derivative constraints. Using this formula, we further derive the effective beamwidth, the maximum range of pointing inaccuracy that can be tolerated to keep the signal suppression in the array output less than a given value. In the GSC, various quiescent beamformers can be used by setting different derivative constraint values. In analyzing the effective beamwidth, we focus on the GSC with the delay-and-sum beamformer as the quiescent beamformer. Such a derivative constraint system can achieve the maximum output SNR when the presteering is perfect, and its performance is independent of the location of the phase origin.

II. BASIC CONFIGURATIONS

Let us consider a K-element equispaced linear array with the narrow-band array response vector defined as

$$a(\psi) = [e^{i\phi_1}, e^{i\phi_2}, \ldots, e^{i\phi_K}]^T$$

(1)

where \(\psi\) is the interelement phase shift, \(\phi_k\) represents the location of the phase origin, and the superscript \(T\) denotes transpose. The adaptive array with look-direction and \(M\) additional constraints can be expressed as the minimization of the array output power

$$\min_w w^*Rw$$

subject to

$$C^*w = f$$

(3)

where the superscript asterisk denotes complex conjugate transpose, \(w\) is the \(K \times 1\) adaptive weight vector, \(R\) is the \(K \times K\) covariance matrix of array inputs, \(C\) is a \(K \times (M + 1)\) constraint matrix, and \(f\) is an \((M + 1) \times 1\) vector whose components are the constraint values. With presteering, the constraints are imposed in the direction of the broad side of the array. Therefore, the constraint matrix implementing the look-direction and the first \(M\) orders derivative constraints can be expressed as

$$C = [\bar{1}, a_1, \ldots, a_M]$$

(4)

where \(\bar{1}\) is a vector of all 1's and \(a_m\) is the \(m\)th derivative vector given by

$$a_m = [(1 - k_0)^m, (2 - k_0)^m, \ldots, (K - k_0)^m]^T.$$  

(5)

Note that \(\bar{1}\) is the equivalent of \(a_0\). Using the binomial expan-
phase origin, $a_m$ of (5) can be rewritten as

$$a_m = \frac{1}{m} \sum_{i=0}^{m} \left( m \right)_{i} (-k_0)^{m-i} \left[ 1, 2, \ldots, M \right]^T.$$  \hfill (6)

From (6), $a_m$ is a linear combination of the $m + 1$ vectors $\left[ 1, 2, \ldots, M \right]^T$, $i = 0, 1, \ldots, m$, each of which is independent of the phase origin $k_0$. The column space of $C$ is spanned by $a_m$'s and hence is independent of $k_0$.

The GSC implementation of the adaptive beamforming described by (2) and (3) is depicted in Fig. 1. In the sidelobe cancelling branch, the signal blocking matrix $B$ is a $K \times (K - M - 1)$ matrix of full column rank and satisfies $C^*B = 0$. Since the column space of $C$ does not depend on the phase origin $k_0$, nor does the signal blocking matrix $B$. As to the quiescent beamformer, it consists of the presteering time delays and the quiescent weights

$$w_q = C(C^*C)^{-1}f.$$  \hfill (7)

Different $f$s will lead to different quiescent beamformers. A quiescent beamformer widely used is the delay-and-sum beamformer as shown in Fig. 1, which will be discussed in Section V. Note that $B^*w_q = 0$ since $B^*C = 0$.

III. SIGNAL SUPPRESSION DUE TO POINTING ERROR

In this section, we will describe the signal suppression phenomenon caused by pointing inaccuracy. To begin with, we assume the array is impinged by the desired signal and is corrupted by white noise. Thus the covariance matrix is written as

$$R = \sigma_s^2 a(\psi)a^*(\psi) + \sigma_n^2 I.$$  \hfill (8)

where $\sigma_s^2$ and $\sigma_n^2$ denote the signal and white noise powers measured at each element, and $\psi$ is the interelement phase shift of the desired signal after presteering. If we achieve perfect pointing accuracy, $\psi = 0$ and $a(\psi) = 1$. Then no signal leaks into the sidelobe cancelling branch and the array gain, the ratio of the output to the input SNR's, achieves the maximum value $K$. However, with pointing error present, i.e., $\psi \neq 0$, the signal will leak into the sidelobe cancelling branch and the leakage tends to cancel the signal output of the quiescent beamformer. In this case, the contribution to the array weight vector from the sidelobe cancelling branch is proportional to $B(B^*B)^{-1}B^*a(\psi)$. If the contribution of the sidelobe cancelling branch contains a component orthogonal to $B(B^*B)^{-1}B^*a(\psi)$, that component will either cause the violation of the constraint equation (3) or increase the output noise power without having any influence on the output of the desired signal. Therefore, the optimal array weight vector can be expressed as

$$w = w_q - \omega B(B^*B)^{-1}B^*a(\psi).$$  \hfill (9)

where $\omega$ is a scalar to be solved later.

From (9), we may define the sidelobe leakage factor as [16]

$$\delta(\psi) = \frac{\|B(B^*B)^{-1}B^*a(\psi)\|^2}{\|a(\psi)\|^2}.$$  \hfill (10)

where $\| \cdot \|$ denotes the euclidean norm of a vector and $\|a(\psi)\|^2 = K$. In physical terms, the sidelobe leakage factor may be viewed as the ratio

$$\delta(\cdot) = \frac{\text{signal leakage power in } B}{\text{signal power received by array}}.$$  

Fig. 1. Generalized sidelobe canceller using delay-and-sum as the quiescent beamformer.

Note that $\delta(\psi)$ defined by (10) is independent of the input signal power and has a value between zero and unity.

In minimizing the output power of (2), it follows from the orthogonality principle that the array output signal is uncorrelated with the sidelobe cancelling signal, i.e.,

$$w^*RB(B^*B)^{-1}B^*a(\psi) = 0.$$  \hfill (11)

Substitution of (9) into (11) yields the solution of $\omega$. Then, from (9), the output signal power $\sigma_s^2\|w^*a(\psi)\|^2$ and the output noise power $\sigma_n^2\|w\|^2$ can be evaluated. For the detail, one may refer to [16], [17].

For the comparison of the array gains of different antenna numbers, we define the normalized array gain as

$$G = \frac{1}{K} \frac{\text{SNR}}{\text{SNR}}$$

where $\text{SNR}_s$ and $\text{SNR}_n$ denote the input and the output SNR's, respectively. Thus, whatever $K$ is, $G$ is not more than unity. From the above discussion, we may obtain the following formula:

$$G_s(\psi) = \frac{G_s(\psi)}{[1 + K \text{SNR}_s \delta(\psi)]^2 + (K \text{SNR}_n)^2 \delta(\psi)G_s(\psi)}$$  \hfill (12)

where $G_s(\psi)$ is the normalized quiescent array gain defined by

$$G_s(\psi) = \frac{\|w^*a(\psi)\|^2}{K\|w\|^2}.$$  \hfill (13)

It can be seen from (12) that the larger $\delta(\psi)$ and/or $K \text{SNR}_s$, the more serious the signal suppression will be. However, (12) does not show quantitatively the array performance due to the complicated nature of the sidelobe leakage factor $\delta(\psi)$ described by (10). In the next section, we shall use the theory of Legendre polynomial to derive a closed-form expression for the sidelobe leakage factor. Therefore, $G(\psi)$ given by (12) can be in closed form and the effective beamwidth can be derived.

The above discussions assume that the array input contains only the desired signal and noise. Under the assumption of perfect presteering, [16] has shown that the first-order constraint leads to negligible influence on the interference suppression if the interference does not fall in the main lobe region. More generally, our numerical results show that derivative constraint...
adaptive arrays with pointing errors can suppress interference located in the sidelobe region with negligible influence on the output SNR.

IV. GSC SIDELobe Leakage Factor

In this section, we will investigate the sidelobe leakage $\delta(\psi)$ caused by small pointing error $\psi$ within the main lobe region. Although the $\delta(\psi)$ functions for the zeroth and first orders, $M = 0, 1$, have been discussed in [16], our formula would be more illustrating and valid for arbitrary order.

The sidelobe leakage factor $\delta(\psi)$ defined by (10) is determined by the signal blocking matrix $B$ and the desired signal response vector $a(\psi)$. It has been discussed in Section II that $B$ does not depend on the phase origin $k_0$. As to $a(\psi)$, the phase origin can be factored out as a complex scaling of $e^{-jk_0\psi}$, which does not affect the squared norm in (10). Therefore, the sidelobe leakage factor is independent of the phase origin.

To evaluate the sidelobe leakage factor, it is helpful to examine the Taylor’s expansion of the desired signal response vector $[17], [18]$

$$a(\psi) = \frac{\sum_{m=0}^{M} (j\psi)^m}{m!} a_m + \frac{(j\psi)^{M+1}}{(M+1)!} a_{M+1} + \cdots.$$  \hspace{1cm} (14)

Substituting (14) into the sidelobe leakage factor given by (10), evidently, the first term on the right-hand side of (14) will be blocked out because

$$B^*a_m = 0, \quad m = 0, 1, \cdots, M.$$  \hspace{1cm} (15)

Therefore, when $\psi$ is small, the $(M+1)$th derivative vector becomes the dominant term and the sidelobe leakage factor can be approximated to

$$\delta(\psi) = \frac{1}{K} \left[ \frac{\psi^{M+1}}{(M+1)!} \right]^2 \|a_{M+1}\|^2.$$  \hspace{1cm} (16)

For convenience, we have denoted

$$a_{M+1} = B(B^*B)^{-1}B^*a_{M+1}$$  \hspace{1cm} (17)

which, from (15), is the residual of $a_{M+1}$ orthogonalized with $\{a_m\}_{m=0}^M$.

It is shown below that $\delta(\psi)$ approximated by (16) preserves the property of being independent of the phase origin $k_0$, a property possessed by $\delta(\psi)$ given in (10). From (6), we have

$$a_{M+1} = [1^{M+1}, 2^{M+1}, \cdots, K^{M+1}]^T + \sum_{m=0}^{M} \left( \frac{M+1}{m} \right) (-k_0)^{M+1-m} [1^m, 2^m, \cdots, K^m]^T.$$ \hspace{1cm} (18)

The second term on the right-hand side is in the column space of $C$ and hence will be blocked out by $B$. Therefore, substituting (18) into (17) yields

$$a_{M+1} = B(B^*B)^{-1}B^*[1^{M+1}, 2^{M+1}, \cdots, K^{M+1}]^T$$ \hspace{1cm} (19)

which is independent of $k_0$. Consequently, $\delta(\psi)$ approximated by (16) is also independent of $k_0$.

Then, we will evaluate the norm of $a_{M+1}$. Since the column spaces of $C$ and $B$ are mutually complementary orthogonal subspaces, we have

$$B(B^*B)^{-1}B^* = I - C(C^*C)^{-1}C^*.$$  \hspace{1cm} (20)

Postmultiplying (20) by $a_{M+1}$ yields

$$a_{M+1} = a_{M+1} - \sum_{m=0}^{M} \mu_m a_m$$ \hspace{1cm} (21)

where $\mu_m$'s are coefficients. Since $a_{M+1}$ is independent of $k_0$, arbitrary $k_0$ can be chosen for the numerical calculation of $\|a_{M+1}\|$. If we choose the phase origin to be the array geometric center, i.e.,

$$k_0 = \frac{1+K}{2}$$

then $a_m$ becomes

$$a_m = \left[ \left( -\frac{K+1}{2} \right)^m, \left( -\frac{K+2}{2} \right)^m, \cdots \right],$$

$$(K-3)^m, \left( -\frac{1}{2} \right)^m$$

which is an odd (even) symmetric vector if $m$ is odd (even). This property is very useful for us to make approximations below.

Therefore, we use $k_0 = (1+K)/2$ in the sequel. $a_m$ shown above can be viewed as a sample vector of $x^m$ within $[-K/2, K/2]$ with the sampling interval of one. Similarly, $a_{M+1}$ of (21) is a sample vector of the $(M+1)$-degree polynomial defined by

$$P_{M+1}(x) = x^{M+1} - \sum_{m=0}^{M} \mu_m x^m.$$  \hspace{1cm} (22)

Therefore, we have the following two approximations

$$\|a_{M+1}\|^2 = \int_{-K/2}^{K/2} |P_{M+1}(x)|^2 dx$$ \hspace{1cm} (23)

and

$$a_{M+1}^H a_{M+1} = \int_{-K/2}^{K/2} x^m P_{M+1}(x) dx.$$ \hspace{1cm} (24)

From (15) and (17), $\{a_m\}_{m=0}^M$ are orthogonal to $a_{M+1}$, and hence we have

$$\int_{-K/2}^{K/2} x^m P_{M+1}(x) dx = 0, \quad m = 0, 1, \cdots, M.$$ \hspace{1cm} (25)

By defining

$$P_{M+1}(x) = P_{M+1}(Kx/2)$$

the integral of (24) can be written as

$$\int_{-1}^{1} x^m P_{M+1}(x) dx = 0, \quad m = 0, 1, \cdots, M.$$ \hspace{1cm} (26)

Since $P_{M+1}(x)$ is a polynomial of degree $M+1$ and satisfies (25), $P_{M+1}(x)$ differs from the $(M+1)$-degree Legendre polynomial, denoted by $L_{M+1}(x)$, only by a constant factor. The Legendre polynomial reads

$$L_{M+1}(x) = \frac{(2M+2)!}{2^{M+1}((M+1)!)} x^{M+1} + \cdots.$$
By comparing the leading coefficients, it follows that
\[ P_{M,+}(x) = \frac{K^{M+1}[(M+1)!]^2}{(2M+2)!} L_{M,+}(x). \] (26)

Using (26), we can evaluate (22) as
\[ \|\mathbf{a}_{M,+}\|^2 = \frac{K}{2M+3} \int_{-1}^{1} |P_{M,+}(x)|^2 dx. \]

Furthermore, by using the property of the Legendre polynomial
\[ \int_{-1}^{1} L_{M,+}(x)^2 dx = \frac{2}{2M+3}, \]
we obtain
\[ \|\mathbf{a}_{M,+}\|^2 = \frac{K}{2M+3} \left[ \frac{K^{M+1}[(M+1)!]^2}{(2M+2)!} \right]^2. \] (27)

Substituting (27) into (16), the sidelobe leakage factor becomes
\[ \delta(\psi) = \frac{(K\psi)^{2M+2}}{2M+3} \left[ \frac{(M+1)!^2}{(2M+2)!} \right]^2. \] (28)

As shown by (28), the increase in the derivative constraint order \( M \) can effectively reduce the signal leakage into the sidelobe cancelling branch.

V. EFFECTIVE BEAMWIDTH

In this section, we derive the maximum range of pointing inaccuracy that can be tolerated by derivative constraint adaptive arrays to keep the signal suppression less than a given value, which can be termed the effective beamwidth. Although the effective beamwidth has been discussed in the literature [3], [6], and [7], their discussions are limited to the arrays with a look-direction constraint only, i.e., \( M = 0 \). Our analysis will be focused on the derivative constraint arrays of any order, i.e., \( M \geq 1 \).

For the GSC, the sidelobe cancelling branch is determined by the constraint order \( M \) and is independent of the phase origin as discussed previously. As to the quiescent beamformer, various weights \( w_\alpha \) can be used by setting different derivative constraint values \( \psi \). Our analysis is focused on the GSC with the delay-and-sum quiescent beamformer as shown in Fig. 1. The performance of such an adaptive array does not depend on the phase origin \( k_0 \) because both the quiescent beamformer and the signal block matrix are independent of the phase origin. As to the GSC with the flat-top quiescent beamformer, which results from setting zero derivative constraint values, the quiescent weights are affected by the location of the phase origin and so does the performance [13]. Furthermore, with the same order of derivative constraints, the use of the delay-and-sum quiescent beamformer achieves the highest output SNR for a perfect presteering. That is because with \( K \) SNR, and \( \delta(\psi) \) fixed, \( G(\psi) \) increases monotonically with \( G_q(\psi) \) as shown by (12), i.e., the larger the output SNR of the quiescent beamformer, the larger the array output SNR. With a perfect presteering, the delay-and-sum quiescent beamformer achieves the maximum quiescent output SNR and, therefore, leads to the maximum array output SNR.

We first examine the normalized array gain of the GSC with the delay-and-sum quiescent beamformer. The associated quiescent weights are all one's, i.e.,
\[ w_\alpha = 1 \] (29)
and the associated derivative constraint values [9] are
\[ \mathbf{f} = C^T \mathbf{1} = [K, a_1^T, \cdots, a_M^T]^T. \] (30)
The quiescent pattern generated by (29) is
\[ G_q(\psi) = \left( \frac{\sin \frac{\gamma}{2} \psi}{K \sin \frac{\gamma}{2} \psi} \right)^2. \] (31)

Substituting the sidelobe leakage factor \( \delta(\psi) \) and the quiescent pattern given by (28) and (31), respectively, into (12), we can obtain a closed-form expression for the normalized array gain \( G(\psi) \) of derivative constraint adaptive arrays of any order. Furthermore, we can approximate \( G(\psi) \) to
\[ G(\psi) = \frac{G_q(\psi)}{1 + 2K SNR, \delta(\psi) + (K SNR)^2 \delta(\psi) G_q(\psi)} \] (32)
since the term concerning \( \delta^2(\psi) \) in the denominator of (12) is quite small within the tolerable range. Based on (32), we will evaluate below the effective beamwidth of derivative constraint arrays.

Given 10 log \( \gamma \) dB as the maximum allowable loss in the output array gain, we are to derive the associated maximum allowable pointing error \( \psi_\alpha \) such that
\[ G(\psi) \geq \frac{1}{\gamma}, \quad |\psi| \leq \psi_\alpha \]
where \( \gamma \) is a value larger than one. It is evident that \( \psi_\alpha \geq \psi_{\alpha,\gamma} \), where \( \psi_{\alpha,\gamma} \) indicates the conventional beamwidth of the delay-and-sum quiescent beamformer, i.e., \( G_q(\psi_{\alpha,\gamma},) = 1/\gamma \). The ratio \( \psi_\alpha/\psi_{\alpha,\gamma} \) can be termed the relative effective beamwidth of adaptive beamforming which is not more than one and will be shown to shrink as \( K \) SNR increases.

Equating the right-hand side of (32) with 1/\( \gamma \) yields
\[ \delta(\psi_\alpha) K SNR, [K SNR, + 2G_q^{-1}(\psi_{\alpha})] + G_q^{-1}(\psi_{\alpha}) = \gamma. \] (33)
To avoid the cross term \( \delta(\psi_\alpha) G_q^{-1}(\psi_{\alpha}) \), we approximate (33) to
\[ \delta(\psi_\alpha) K SNR, [K SNR, + 2] + G_q^{-1}(\psi_{\alpha}) = \gamma. \] (34)
The reason for (34) being a good approximation of (33) is explained below.

When \( K \) SNR, > 1, since \( 1 \leq G_q^{-1}(\psi_{\alpha}) \leq \gamma, 2G_q^{-1}(\psi_{\alpha}) \) in the brackets of (33) is negligible as compared to \( K \) SNR, and hence can be approximated as in (34). When \( K \) SNR, \( < 1 \), \( \delta(\psi_\alpha) K SNR, \) is quite small and, hence, both (33) and (34) can reduce to \( G_q^{-1}(\psi_{\alpha}) = \gamma \). Since (34) is consistent with (33) for \( K \) SNR, > 1 and \( K \) SNR, < 1, it is reasonable to assume that (34) is also an acceptable approximation of (33) for median \( K \) SNR, because both the \( \psi_\alpha \)'s given by (33) and (34) are smooth curves versus \( K \) SNR.

Then we substitute (28) into (34) to solve \( \psi_\alpha \). We first observe from (28) that \( \delta(\psi_\alpha) \) is proportional to \( |\phi_q^{M+1}|^2 \). Therefore, in order to yield a closed-form solution for (34), we can approximate \( G_q^{-1}(\psi_{\alpha}) \) in (34) by a polynomial containing \( |\phi_q^{M+1}| \) and a constant term to fit (34) in a quadratic equation of \( |\phi_q^{M+1}| \). With the constraints \( G_q^{-1}(0) = 1 \) and \( G_q^{-1}(\psi_{\alpha,\gamma}) = \gamma \), we approximate
From (28), (34), and (35), we obtain the relative effective beamwidth of the derivative constraint adaptive array, \( M \geq 1 \),

\[
\frac{\psi_{\text{rel}}}{\psi_{\text{q,rel}}} = \frac{2}{1 + \sqrt{1 + 4 \Lambda (K\psi_{\text{q}})^2 \Gamma^2}} \frac{1}{M+1},
\]

(36)

where

\[
\Lambda = \frac{1}{2M+3} \left( \frac{(M+1)!}{(2M+2)!} \right)^2,
\]

and

\[
\Gamma = (K\text{SNR})^2 + 2K\text{SNR}_i.
\]

The relative half-gain effective bandwidth is especially of interest. In this case, \( \gamma = 2 \) and \( \psi_{\text{q,rel}} = 0.886 \pi/K = 2.78/K \).

Direct substitution of \( \psi_{\text{q,rel}} \) into (36) yields

\[
\frac{\psi_{\text{rel}}}{\psi_{\text{q,rel}}} = \frac{2}{1 + \sqrt{1 + 4 \Lambda \Gamma^2 2.78^2}} \frac{1}{M+1},
\]

(37)

From (36) and/or (37), some remarks can be made. When \( K\text{SNR}_i \) or equivalently \( \Gamma \) is very low, the relative effective beamwidth is about one, i.e., the adaptive array can preserve the quiescent beamformer beamwidth \( \psi_{\text{q,rel}} \) as the effective beamwidth \( \psi_{\text{rel}} \). As \( K\text{SNR}_i \) increases, the denominator of (36) increases and hence the effective beamwidth decreases.

VI. NUMERICAL RESULTS

Numerical results are presented to confirm the analysis for the equispaced linear arrays with derivative constraints. Fig. 2 plots the sidelobe leakage factor for pointing inaccuracy within the main lobe region with \( K = 8 \) and \( 20 \). The solid lines are the associated approximations given by (28). We find that for small pointing error \( K\phi \), the larger the antenna number \( K \), the more accurate the approximation (28) will be. When the pointing error is large, the discrepancy between the approximation and the actual \( \delta(\phi) \) is majorly due to the truncation of the Taylor’s series because the \( (M+2) \)th derivative \( \psi^{M+2}_{\text{rel}} \) in (14) becomes unnegligible as compared to the \( (M+1) \)th derivative \( \psi^{M+1}_{\text{rel}} \). Therefore, it follows from (16), (28), and the Taylor’s theorem that the ratio of the approximation error to the actual \( \delta(\phi) \) would be in the order of \( O((K\phi)^2) \). In Fig. 3, we present the array main lobe bearing response to a signal with \( K\text{SNR}_i = 100 \) for the approximation given by (32). For comparison, the array response of the GSC with the flat-top quiescent beamformer is also presented in Fig. 3(b). As shown, when \( M \geq 2 \), the flat-top method achieves the main lobe broadening at the price of losing at least 3 dB in output array gain when the presteering is perfect. Besides, the use of the delay-and-sum quiescent beamformer can yield a higher output array gain than the flat-top method within a considerable range. In Fig. 4, the relative 3 and 6 dB effective beamwidths and the associated approximations given by (36) are plotted versus \( K\text{SNR}_i \). It is evident that the larger \( K\text{SNR}_i \), the more accurate the beam steering must be, otherwise the constraint order \( M \) should be increased.

In Fig. 5, we plot the array output SINR with interference.
K = 20

\[
I = \log(K \text{ SNR})
\]

Fig. 4. Relative effective beamwidth of the GSC with the delay-and-sum quiescent beamformer. (a) 3 dB, i.e., half-gain. (b) 6 dB.

present. We assume an array with \( K = 12 \) presteered to a desired signal of \( K \text{ SNR} = 100 \) with a pointing error \( \psi/2\pi = 0.4 \), and let an interferer of equal strength vary as the abscissa. The horizontal straight lines in Fig. 5 are the array output SNR's when the interferer is absent. As shown, an interferer in the sidelobe region can be well suppressed so that the array output SINR is almost the same as the output SNR of the interference-absent case.

VII. CONCLUSION

We have investigated the effects of pointing inaccuracy on derivative constraint adaptive arrays based on the GSC model. Under the assumption of equispaced linear arrays, we have utilized the theory of Legendre polynomial to derive a closed-form formula to predict the sidelobe leakage factor. Furthermore, we have derived the effective beamwidth of the derivative constraint GSC adaptive array with the delay-and-sum quiescent beamformer. The performance of such a derivative constraint system is independent of the phase origin and can achieve the maximum output SNR when the presteering is perfect. Numerical results validate the analysis, especially when the number of array elements is large.

REFERENCES

[15] N. K. Jablon, "Adaptive beamforming with the generalized side-
Abstract—A monolithic azimuthal monopulse antenna for 94 GHz applications has been developed. The structure consists of a single dipole suspended in one plane of an integrated horn cavity to obtain the sum pattern, and an antiparallel pair of dipoles suspended in a different plane of the same horn cavity to achieve the difference pattern. Pattern measurements of microwave models and on the millimeter-wave antennas show good agreement with theory and exhibit symmetry with a sharp -30 dB null at broadside for the difference antenna. Microwave model measurements show input impedances close to 50 $\Omega$ with greater than -25 dB isolation between sum and difference antennas across a 10% bandwidth.

I. INTRODUCTION

Monopulse direction-finding techniques are currently the most accurate and rapid method for locating a target electronically [1]. A simple monopulse system combines information received by two channels: the “sum” ($\Sigma$) and “difference” ($\Delta$) channels. These produce broadside patterns with a broad peak and sharp null, respectively, and allow a system to obtain information about the position of a target. There are two commonly used methods to generate the monopulse patterns [1]–[5]. The first is to excite the appropriate modes within a single feed horn. The second method is to microstrip or waveguide antenna arrays coupled together using external combining networks. The former technique is used in the system presented here. This is advantageous because an external combining network can be quite lossy at millimeter-wave frequencies and is complex and expensive.

The monopulse antenna presented in this paper is based on the monolithic millimeter-wave horn cavities developed by Rebeiz and Rutledge [6], [7] using pyramidal cavities etched in silicon. The integrated horn antennas currently have the highest efficiency of any monolithic antenna at millimeter-wave frequencies [8]. The monopulse system achieves direction finding in a single coordinate (azimuth) using two separate antennas within the same horn cavity (Figs. 1 and 2). The sum antenna consists of a single dipole and is suspended on a dielectric membrane with a thickness much less than a wavelength. This antenna couples primarily to the $TE_{10}$ mode, which produces a broad peak at normal incidence. Similarly, a set of parallel dipoles is suspended within the cavity on a membrane at a different plane. These dual dipoles, which constitute the difference antenna, are connected together by coplanar strip transmission lines which are crossed over at the center to couple primarily to the $TE_{20}$ mode of the cavity. This mode produces a pattern which contains a sharp null at normal incidence. The detector is integrated at the center of the membrane where the two transmission lines meet. The construction of this monopulse system is achieved using photolithographic techniques identical to those used in the production of conventional integrated circuits. The compact construction of the antenna makes it highly suitable for use in numerous aviation and navigation applications.

II. FABRICATION

The integrated monopulse antenna has been fabricated at the University of Michigan. The antenna is composed of a stacked silicon wafer structure. The thicknesses of the front wafer is chosen to position the difference antenna at the proper depth within the horn cavity. The wafers stacked below the difference antenna position the sum antenna at the proper depth. Finally, wafers are stacked below the sum antenna to complete the pyramidal horn cavity.

The horn cavity is made by anisotropic etching of silicon wafers with a $(100)$ crystallographic orientation. This results in the formation of a pyramidal hole bounded by the $(111)$ crystal planes [9]. The orientation of the crystal planes constrains the flare angle of the horn cavity to be $70.6^\circ$. The membranes are composed of oxides and nitrides grown or deposited on the silicon and designed to have a small amount of tensile stress. This stress allows the membrane to be flat and self-supporting. The silicon is then etched from behind the membrane at the location of the cavity holes. After etching, antennas and detectors are deposited on the membrane using conventional photolithographic methods. The detectors used here are $4 \mu m$-square bismuth microbolometers. Recent work has resulted in a design that includes a hybrid-mounted beam-lead Schottky diode for heterodyne detection [12]. Finally, the individual wafers are assembled together by aligning and gluing them.

III. THEORY

The theoretical patterns of the monopulse configuration are easily obtained by modeling the horn cavity by a cascade of very small waveguide steps, and using mode-matching theory. The dipole antennas within the cavity are modeled by Hertzian dipoles and the resulting fields at the horn mouth are calculated. A detailed analysis is presented in [10]. The difference antenna